Supplementary Material for “Measuring the frequency dynamics of financial connectedness and systemic risk”*

Jozef Baruník† and Tomáš Křehlík

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1 Supplementary Tables and Figures

1.1 Including Fannie Mae (FNM) and Freddie Mac (FRE)

As suggested in the data section, FRE and FNM were, for a substantial portion of the period, part of the US financial system. Hence, as a robustness measure, we compute the overall connectedness with the same specification over the shorter dataset when both FRE and FNM are publicly traded and compare the overall connectedness of the two systems to see how the results change. The result is shown in Figure [1]. We can see that most of the time, the system that includes the FRE and FNM was more connected than was the restricted system. The difference peaks at approximately 6% in mid-2006, which is a relatively high number due to the nature of the connectedness, which is a mean of shares of variances created by shocks to other variables. Hence, including FRE and FNM in the system increases the systemic risk although the dynamics do not substantially differ from the analysis that omits FRE and FNM.

*For estimating the frequency-dependent connectedness measures introduced by this paper, we provide the package frequencyConnectedness in R software. The package is available on CRAN or https://github.com/tomaskrehlik/frequencyConnectedness.

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Figure 1: Time dynamics of connectedness of the US financial sector for system with and without FNM and FRE. The top part shows the level of connectedness and the bottom part shows the difference. The horizontal axis shows time. A positive difference indicates that the system including FNM and FRE is more connected than the system without those two stocks.
1.2 Supplementary Tables and Figures

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Table 1: Simulation results. The first three columns describe parameters for the simulation as described in Equation (??). The estimate is computed averaging over the 1000 simulations of VAR with length $10^4$, and the standard error is the sample standard deviation.
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**Table 2:** The true values for connectedness in the VAR settings.

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**Table 3:** The descriptive statistics of the volatility data
Figure 2: The decomposition of connectedness with cross-sectional dependence. The individual lines represent connectedness measures at a given frequency band, more concretely: i) connectedness from one day to one week, ii) connectedness from one week to one month, and iii) connectedness from one month to 300 days. The shaded area represents the space between the 5% and 95% quantiles of the bootstrapped measure.
Figure 3: The decomposition of connectedness \textit{without} cross-sectional dependence. The individual lines represent connectedness measures at a given frequency band, more concretely: \textit{i)} connectedness from one day to one week, \textit{ii)} connectedness from one week to one month, and \textit{iii)} connectedness from one month to 300 days. The shaded area represents the space between the 5\% and 95\% quantiles of the bootstrapped measure.