Supplementary Material for "Total, asymmetric and frequency connectedness between oil and forex markets"

Jozef BARUNÍK*and Evžen KOČENDA

A Realized variance and semivariance

In this Section we briefly introduce realized measures that we use for volatility connectedness estimation. We begin with realized variance and then we describe realized semivariances. Realized measures are defined on a continuous-time stochastic process of log-prices, p_t, evolving over a time horizon $[0 \leq t \leq T]$. The process consists of a continuous component and a pure jump component,

$$p_t = \int_0^t \mu_s ds + \int_0^t \sigma_s dW_s + J_t, \tag{1}$$

where μ denotes a locally bounded predictable drift process, σ is a strictly positive volatility process, and J_t is the jump part, and all is adapted to some common filtration $\mathcal{F}$. The quadratic variation of the log prices p_t is:

$$[p_t, p_t] = \int_0^t \sigma_s^2 ds + \sum_{0<s\leq t} (\Delta p_s)^2, \tag{2}$$

where $\Delta p_s = p_s - p_{s-}$ are jumps, if present. The first component of Eq. (2) is integrated variance, whereas the second term denotes jump variation. Andersen and Bollerslev (1998) proposed estimating quadratic variation as the sum of squared returns and coined the name "realized variance" (RV). The estimator is consistent under the assumption of zero noise contamination in the price process.

Let us denote the intraday returns $r_k = p_k - p_{k-1}$, defined as a difference between intraday equally spaced log prices $p_0, \ldots, p_n$ over the interval $[0, t]$, then

$$RV = \sum_{k=1}^{n} r_k^2 \tag{3}$$

*Corresponding author, Tel. +420(776)259273, Email address: barunik@fsv.cuni.cz

converges in probability to $[p_t, p_t]$ with $n \to \infty$.

Barndorff-Nielsen et al. (2010) decomposed the realized variance into realized semivariances (RS) that capture the variation due to negative (RS^-) or positive (RS^+) price movements (e.g., bad and good volatility). The realized semivariances are defined as:

$$RS^- = \sum_{k=1}^{n} \mathbb{I}(r_k < 0) r_k^2, \tag{4}$$

$$RS^+ = \sum_{k=1}^{n} \mathbb{I}(r_k \geq 0) r_k^2. \tag{5}$$

Realized semivariance provides a complete decomposition of the realized variance, hence:

$$RV = RS^- + RS^+. \tag{6}$$

The limiting behavior of realized semivariance converges to $1/2 \int_0^t \sigma_s^2 ds$ plus the sum of the jumps due to negative and positive returns (Barndorff-Nielsen et al., 2010). The negative and positive semivariance can serve as a measure of downside and upside risk as it provides information about variation associated with movements in the tails of the underlying variable.

References

Andersen, T. G. and T. Bollerslev (1998). Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *International economic review*, 885–905.

Barndorff-Nielsen, O., S. Kinnebrock, and N. Shephard (2010). *Volatility and Time Series Econometrics: Essays in Honor of Robert F. Engle*, Chapter Measuring Downside Risk-Realised Semivariance. Oxford University Press.

B Supplementary Tables and Figures

B.1 Directional Connectedness

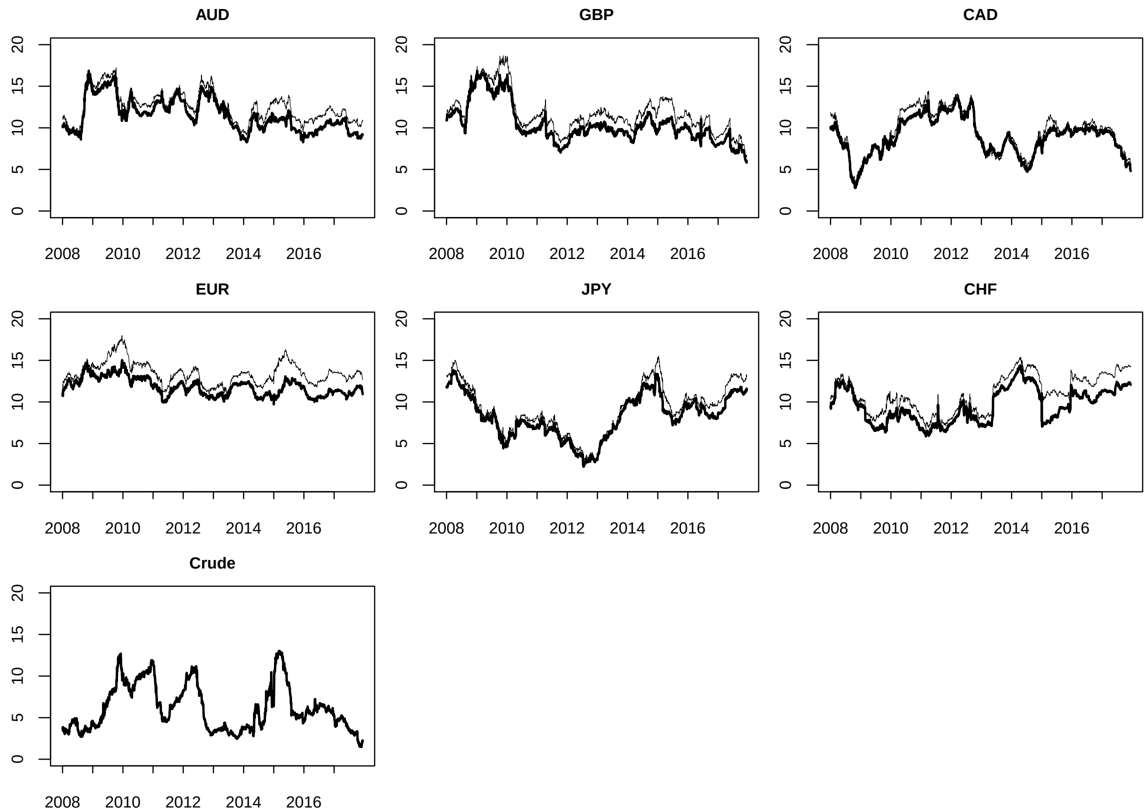

Figure 1: The directional (TO) volatility connectedness of six currencies (solid line), the directional volatility connectedness of six currencies and crude oil (bold solid line). The directional (TO) volatility connectedness quantifies how volatility from a specific asset (oil or currency) transmits to other assets in portfolio ("contribution TO").

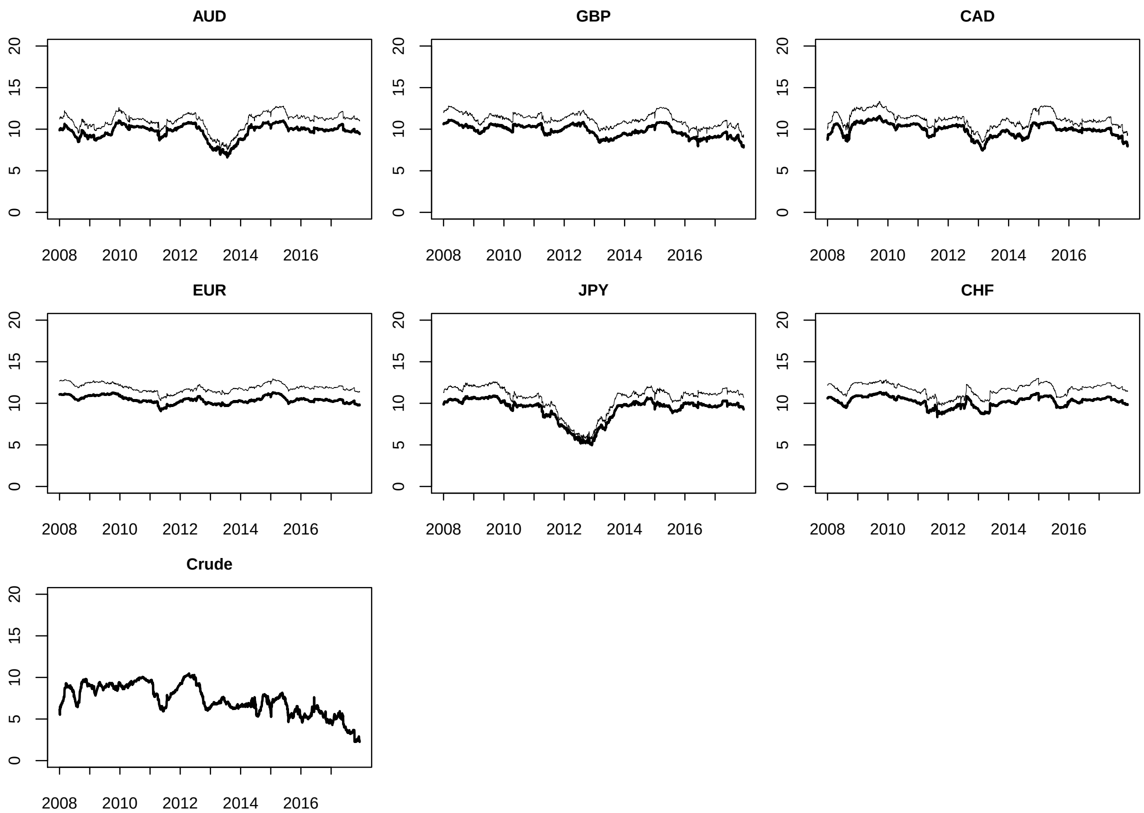

Figure 2: The directional (FROM) volatility connectedness of six currencies (solid line), the directional volatility connectedness of six currencies and crude oil (bold solid line). The directional (FROM) volatility connectedness quantifies how volatility from a group of assets transmits to a specific asset (oil or currency) ("contribution FROM").

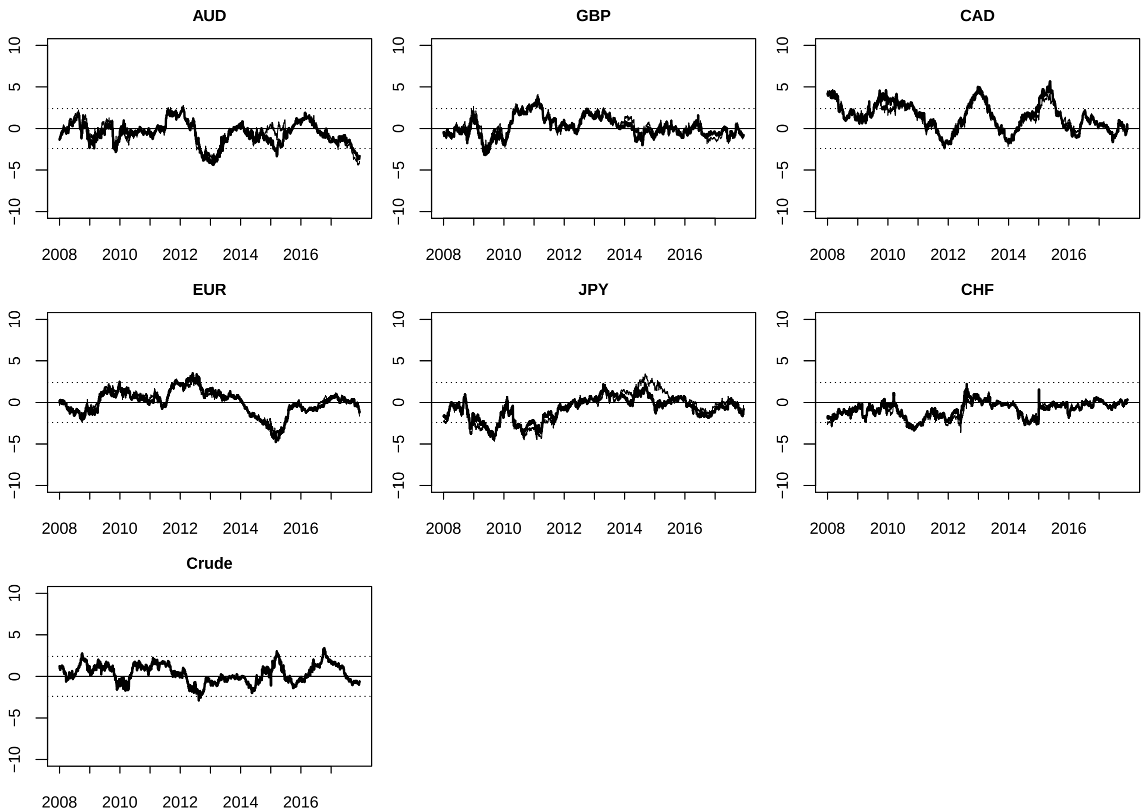

Figure 3: Directional (TO) Spillover asymmetry measure (SAM). Solid line represents the directional SAM for the forex market only, while the bold solid line represents the directional SAM for the crude oil and forex markets. 95% bootstrapped confidence bands are shown by dotted lines. The directional SAM (TO) quantifies how asymmetry in volatility from a specific asset (oil or currency) transmits to other assets in portfolio ("contribution TO"

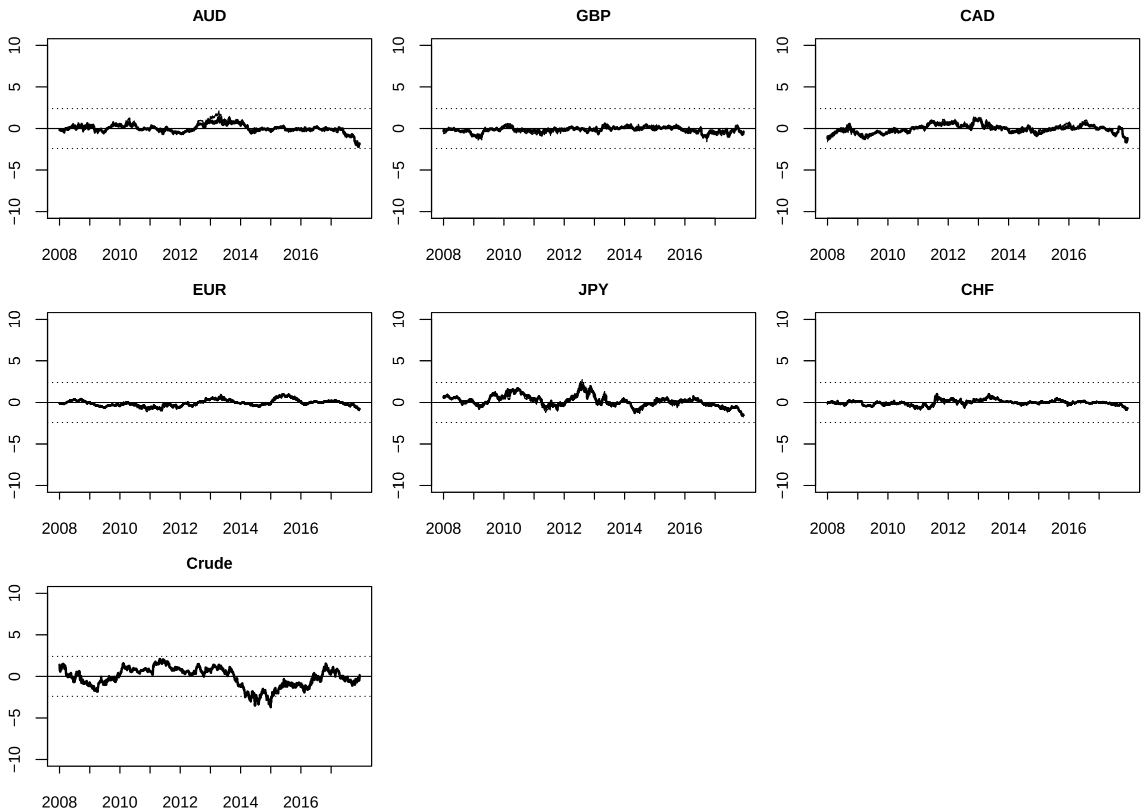

Figure 4: Directional (FROM) Spillover asymmetry measure (SAM). Solid line represents the directional SAM for the forex market only, while the bold solid line represents the directional SAM for the crude oil and forex markets. 95% bootstrapped confidence bands are shown by dotted lines. The directional SAM (FROM) quantifies how asymmetry in volatility from a group of assets transmits to a specific asset (oil or currency) ("contribution FROM").

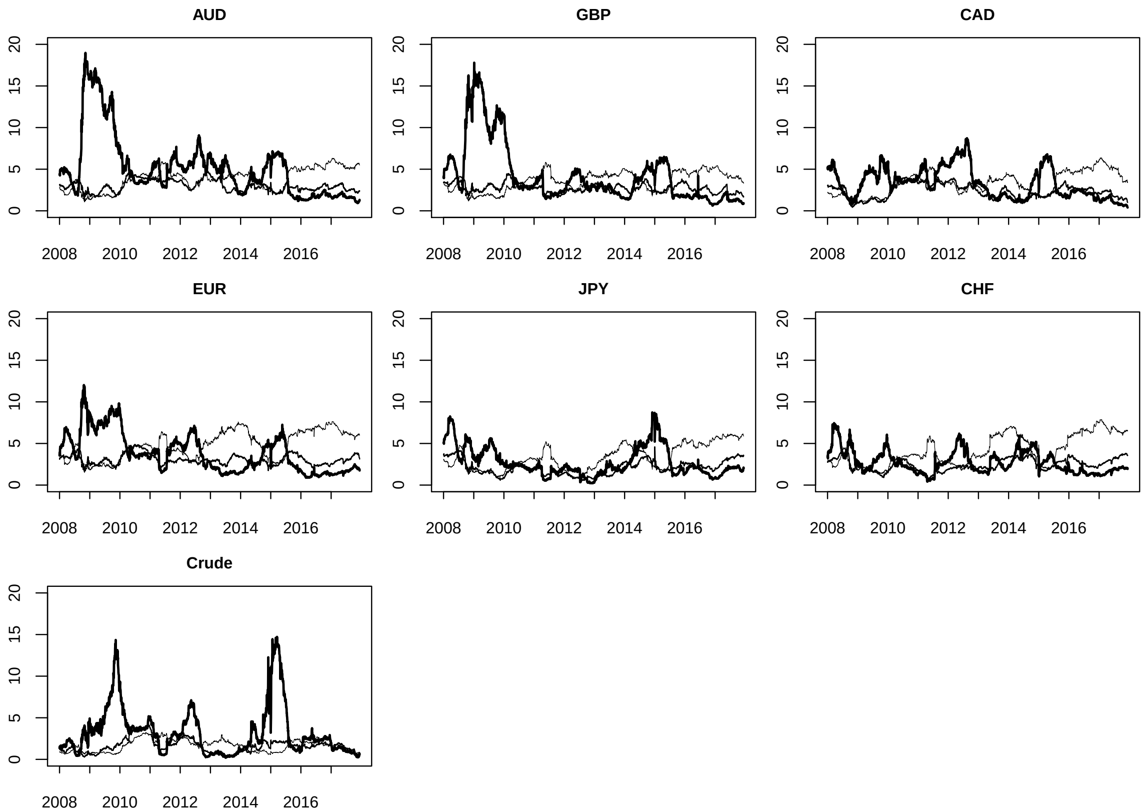

Figure 5: Directional (TO) frequency connectedness. The frequency connectedness at short-term horizon defined at $d_1 \in [1, 5]$ days in solid line, medium-term horizon defined at $d_2 \in (5, 20]$ days medium bold line, and long-term horizon defined at $d_3 \in (20, 300]$ days in bold line. Note that all lines through the frequency bands d_s sum to the total connectedness. The directional (TO) frequency connectedness quantifies how volatility (measured at given frequency) from a specific asset (oil or currency) transmits to other assets in portfolio ("contribution TO").

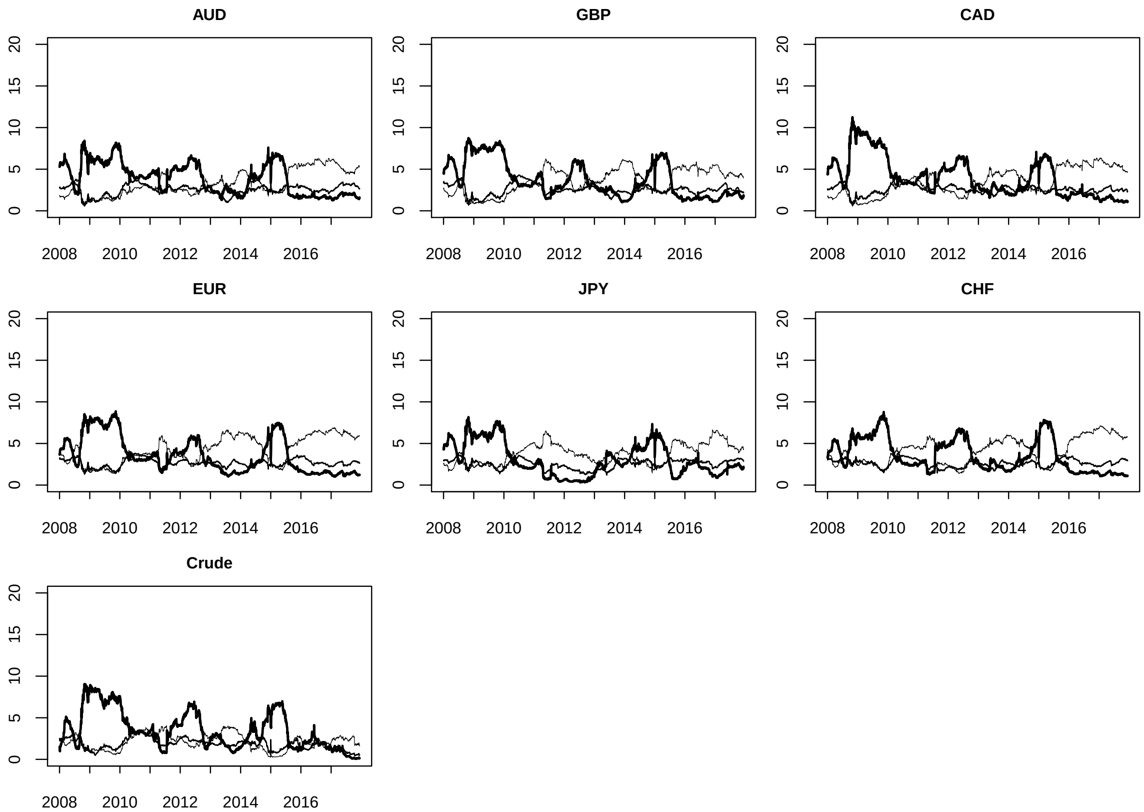

Figure 6: Directional (FROM) frequency connectedness. The frequency connectedness at short-term horizon defined at $d_1 \in [1,5]$ days in solid line, medium-term horizon defined at $d_2 \in (5,20]$ days medium bold line, and long-term horizon defined at $d_3 \in (20,300]$ days in bold line. Note that all lines through the frequency bands d_s sum to the total connectedness. The directional (FROM) frequency connectedness quantifies how volatility (measured at given frequency) from a group of assets transmits to a specific asset (oil or currency) ("contribution FROM").

B.2 Sensitivity Analysis

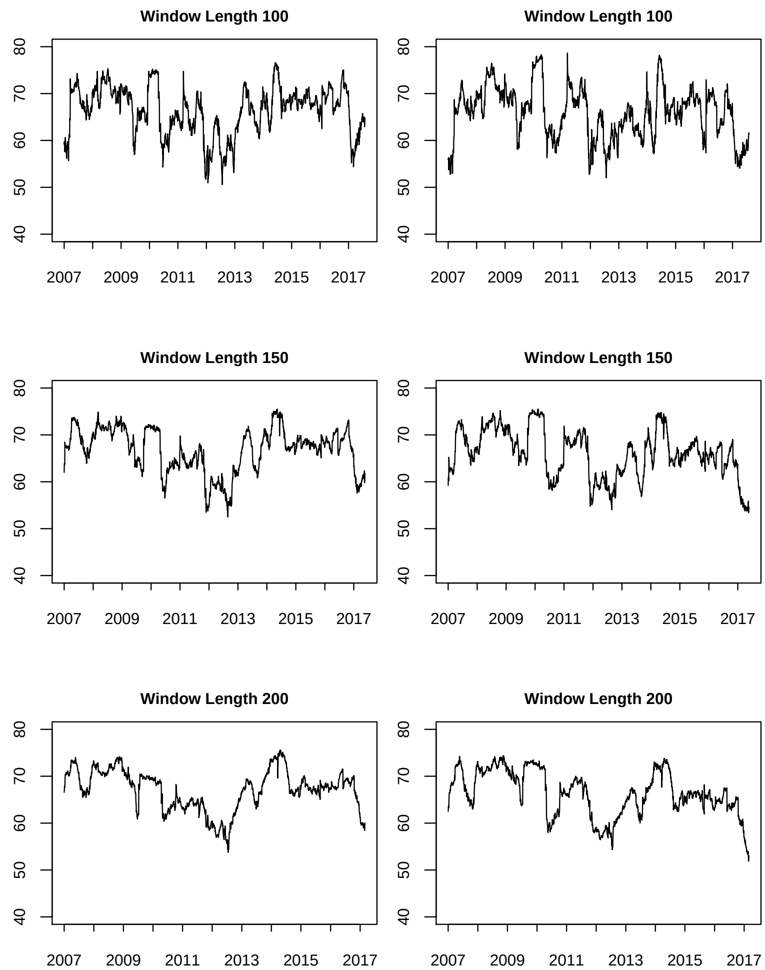

Figure 7: Sensitivy Analysis: Total volatility connectedness of six currencies (left column), and of six currencies and crude oil (right column). Total connectedness computed for different window lengths (top row), horizons (middle row), and VAR lengths (bottom row).

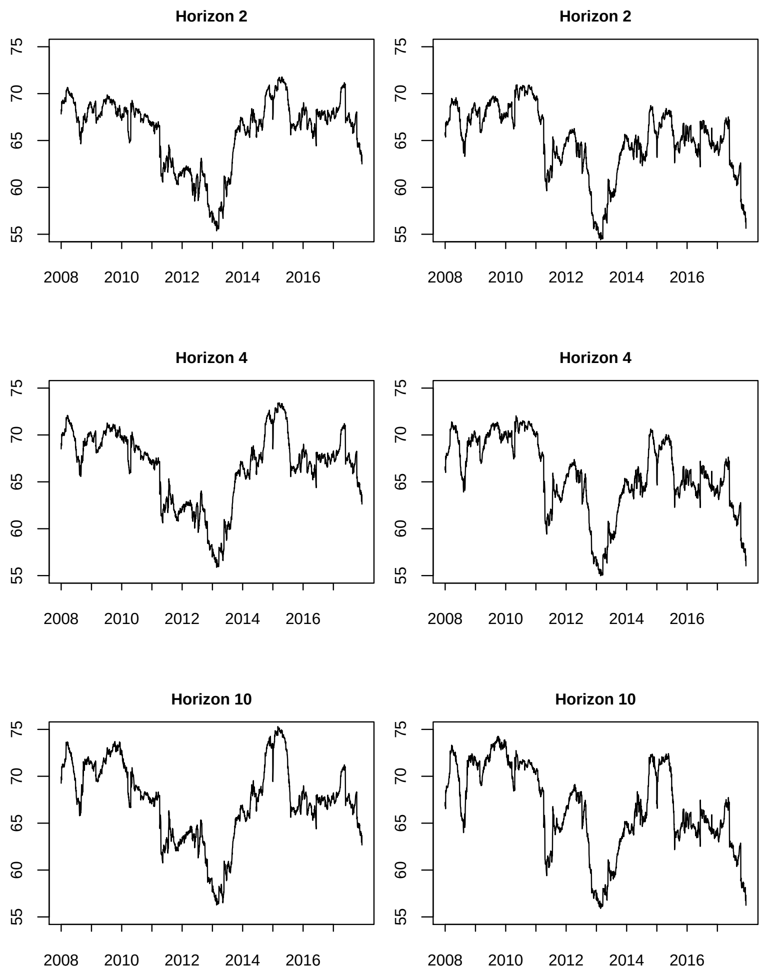

Figure 8: Sensitivy Analysis: Total volatility connectedness of six currencies (left column), and of six currencies and crude oil (right column). Total connectedness computed for different window lengths (top row), horizons (middle row), and VAR lengths (bottom row).

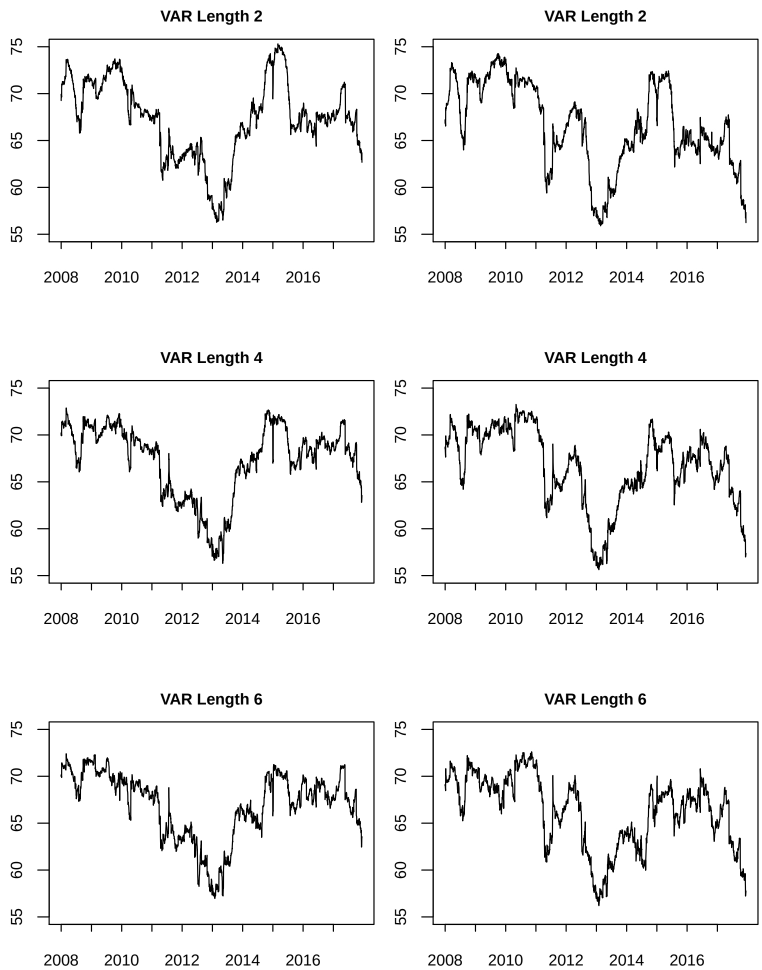

Figure 9: Sensitivy Analysis: Total volatility connectedness of six currencies (left column), and of six currencies and crude oil (right column). Total connectedness computed for different window lengths (top row), horizons (middle row), and VAR lengths (bottom row).